ORIGINAL ARTICLE

Applicability of mathematical curve-fitting procedures to late mixed dentition patients with crowding: A clinical-experimental evaluation

Hans Wellens
Brugge, Belgium

Introduction: This study aimed at investigating the applicability of a polynomial function laterally, combined with a parabola or hyperbolic cosine function in the front, for mandibular curve-fitting purposes in late mixed dentition patients with crowding. The possible advantages of using a bilateral center of gravity for curve construction were examined. **Methods:** After digitizing 30 mandibular casts (14 boys, 16 girls), 14 coordinates per model were used to fit a hyperbolic cosine function or a parabola in the anterior segment and a third-degree polynomial function bilaterally, by using the method of least squares. The lateral functions were fitted by using a mirrored center of gravity for the premolar/molar area. To assess curve fit and enable comparison with other studies, Pearson correlation coefficients, residuals, mean square error, root mean square values, and average perpendicular distance of the points to the constructed curve were calculated. **Results and Conclusions:** High correlation coefficients were found (mean, 0.994; SD, 0.004). The mean square error (1.80 mm^2, SD, 1.12 mm^2) and the average root mean square value (1.28 mm, SD, 0.42 mm) appears to be comparable with other studies. Combined with the relatively low average perpendicular distance of the points to the curve (mean, 0.47 mm; SD, 0.16 mm), it seems that this method is successful in describing mandibular arch forms of mixed dentition patients with crowding, and it could be a useful tool in treatment planning. (Am J Orthod Dentofacial Orthop 2007;131:160.e17-160.e25)

Crowding is the practical expression of a tooth size-arch length discrepancy (TSALD): too great a difference between the accumulated mesiodistal tooth width of mainly the anterior dentition and the space available to accommodate these teeth in a nicely aligned fashion can lead to rotations and buccal or lingual displacements. Many approaches have been proposed to correct crowding: incisor proclination or protrusion,[1] expansion of the lateral segments,[2] molar distalization,[3] use of leeway space,[4] interproximal stripping,[5] extraction therapy, and, more recently, distraction osteogenesis at the mandibular midline,[6] all of which aim at harmonizing the relationship between tooth size and the available supporting alveolar bone.

Because of the great variety of these treatment possibilities, assessing the amount of crowding is an important part of any orthodontic diagnosis. A detailed knowledge of the patient's TSALD allows the practitioner to make well-founded treatment decisions, based on precise, goal-oriented treatment planning.[7] This is especially true for borderline extraction and nonextraction patients.

To obtain a specific millimetric measurement of the required and available space for incisor alignment, 2 parameters should be calculated: combined anterior tooth width and available arch length.[7] Whereas the first is relatively easy to establish, determining the latter can be challenging, because available arch length is influenced by several treatment factors: changes in intercanine and intermolar widths,[8-10] incisor protrusion[8-10] and torque,[11,12] and changes in posterior arch depth (ie, molar distalization). Furthermore, the space needed for incisor alignment is equally influenced by changes in incisor angulation[11] and inclination,[11,12] tooth size discrepancies,[13] incisor reproximation,[14] and, to some extent, even the correction of the curve of Spee.[15,16]

With so many influencing factors, accurately establishing the available space for incisor alignment requires the practitioner to have a well-defined concept of the arch form the patient will be treated to. For

Private practice, Brugge, Belgium.
Supported by a grant from the Scientific committee of the Society of Flemish Dentists (Verbond der Vlaamse Tandartsen).
Reprint requests to: Hans Wellens, Groene-Poortdreef 16, 8200 Sint-Michiels, Brugge, Belgium; e-mail, wellens.hans@telenet.be.
Submitted, March 2006; revised and accepted, June 2006.
0889-5406/$32.00

doi:10.1016/j.ajodo.2006.06.015

treatment-planning purposes, the orthodontist should therefore ideally have full control over all parameters that influence arch form, such as intercanine and intermolar widths, anterior and posterior arch lengths, and lateral arch curvature.

Through the years, many articles demonstrated various ways to mathematically describe the human arch form. Reported methods range from simple geometric forms such as parabola[17] and ellipse[18] to geometric functions such as catenary curves,[15,19] polynomials,[19,20] cubic spline functions,[21] conic sections,[22] the beta function,[23] combinations of the hyperbolic cosine function and beta function,[9] and Fourier analysis.[24] From an experimental and scientific standpoint, many studies use only well-aligned dentitions with good occlusions to test the descriptive method of interest,[17-21] because this is the most efficient way to demonstrate the effectiveness of the mathematical curve description. However, because most patients have some TSALD, it is of interest to investigate the mathematical description of arch form in patients with mandibular labial crowding.

A recent study demonstrated extremely accurate reproduction of arch form in patients with seriously malpositioned teeth with Fourier analysis.[24] The resulting curve represents a true reproduction of the patient's arch form with all its peculiarities. However useful this may be for studying arch form and shape changes or for measuring crowding, for treatment planning purposes it would be interesting to assess the applicability of curve-fitting procedures used to describe patients with ideal alignment. This would require the construction of an "optimal" individualized arch form: the specific arch form we would like to establish in a particular patient. This would enable a more accurate assessment of available arch length. Also, as Brennan and Gianelly[25] suggested that mandibular crowding can be resolved relatively easily in the late mixed dentition by using the average 4.4 mm of combined bilateral leeway space, treatment planning should accommodate the accurate measurement of TSALD at this stage of dental development.

It was the aim in this study to investigate the applicability of a third-degree polynomial function laterally, combined with a parabola or hyperbolic cosine function in the front for mandibular curve-fitting procedures in late mixed dentition patients with crowding. The possible advantages of multiple centers of gravity was examined.

MATERIAL AND METHODS

Thirty subjects, 14 boys and 16 girls, were selected from 3 private orthodontic practices with the following inclusion criteria: (1) all permanent incisors, canines, and first premolars fully erupted; (2) at least 1 deciduous second molar present; (3) incisors with fairly normal buccolingual inclinations, according to visual inspection; (4) no permanent teeth in crossbite; (5) no clear ectopic or tipped canines; (6) fairly symmetric occlusion; (7) no previous orthodontic treatment; (8) casts in good condition.

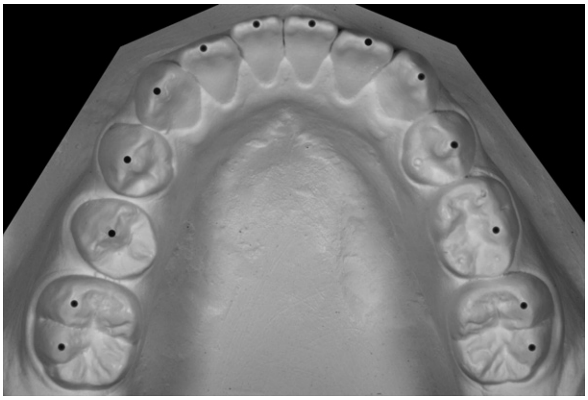

Fig 1. Landmarks used in study.

All mandibular models were trimmed so that, when placed with the occlusal surface on a flat plane, the first molars and incisors were in contact with the table. A reference plane was thus constructed through the distobuccal cusp tips of the first molars and the incisors. Subsequently, the following 16 points were marked with a fine ink pen (Rotring rapidograph 0.18, Rotring, Hamburg, Germany) (Fig 1): middle of the incisal edge of the central and lateral incisors, cusp tip of the canines, buccal cusp tip of the first premolars, midbuccal cusp tip of the second deciduous molar or buccal cusp tip of the second premolar, and mesiobuccal and midbuccal cusp tip in case of 3 buccal cusps, or the mesiobuccal and distobuccal cusp tip in case of 2 buccal cusps, of the first definitive mandibular molars.

To assess irregularity, Little's irregularity index[26] was determined by measuring the contact point displacements from the left to the right mandibular canine. The 5 values were added to give the irregularity index.

Next, the models were placed on the scanning surface of a desktop scanner (Scanjet 8200, Hewlett Packard, Palo Alto, Calif) as follows: a transparent calibration sheet marked with 3 points was placed approximately in the center of the scanning surface. On top, the model was positioned, with the occlusal surface downward. Paperweights were added in the x and y directions, ensuring that the calibration sheet was flat against the scanner surface. Scanning was performed at a resolution of 2400 dpi.

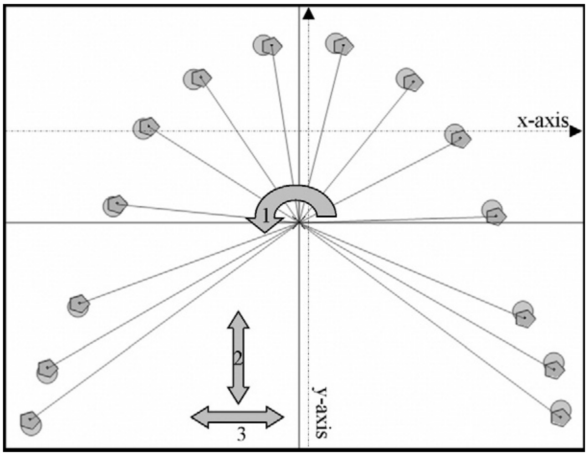

Fig 2. Orientation of digitized points. Digitized landmarks are depicted as pentagons. In middle, COG is located. First, angle between line connecting each landmark with COG and horizontal through this COG is calculated. Fourteen angles are averaged, and resulting value is used to rotate cluster of points around COG, so new average angle equals zero. Resulting positions are shown as *circles*. Vertically, canine tips are already centered around y-axis. Cluster of points is shifted horizontally, so COG coincides with y-axis (*vertical dotted line*).

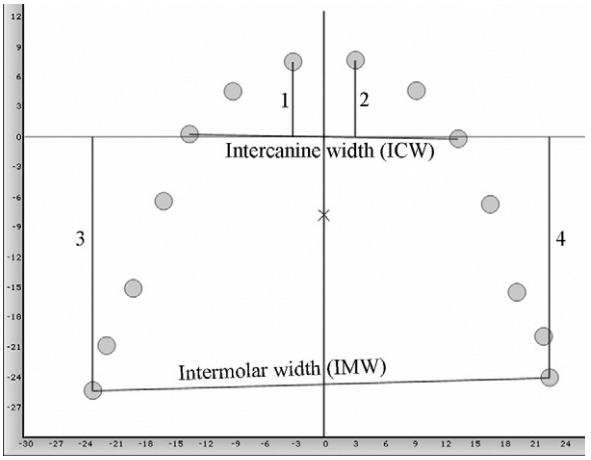

Fig 3. Calculated curve-fitting parameters. Intercanine width, straight line distance between canine cusp tips. Intermolar width, straight line distance between distobuccal or midbuccal cusp tips, depending on presence of 2 vs 3 mandibular first molar cusp tips, respectively. Anterior arch depth, AAD = (1+2)/2. Posterior arch depth, PAD = (3+4)/2. Total arch depth, AAD + PAD. Averaged canine coordinates, (−ICW/2, AAD); (ICW, AAD). Averaged molar coordinates, (−IMW/2, −PAD); (IMW, −PAD).

The images were then imported into a digitizing software program (DigitizeIt 1.5.7, I. Bormann, Bormisoft, Braunschweig, Germany). With this software, it is possible to determine the x and y coordinates of selected points after clicking the 3 markers of the calibration sheet, defining the orthogonal axis system. Because scanning was performed at 2400 dpi, 1 pixel on the computer screen had a true dimension of 0.011 mm. In this way, all 16 landmarks were digitized and oriented (Fig 2).

Next, the curve-fitting parameters were calculated as described in Figure 3. The values were entered into Findgraph (FindGraph for Windows, version 1.482, UNIPHIZ Lab, 2001-2004, Tver, Russia), a curve-fitting and graphing program. The author of this software program compiled a dedicated plug-in, automating the curve-fitting procedure based on the method of least squares. In accordance with the work of Hnat et al,[9] the mandibular arch form was described by 3 different functions linked together, as shown in Figure 4. To adequately describe the lateral segments, a bilateral center of gravity (COG) was calculated (Fig 4). In constructing the curves, the clinician should optimize the fit of the curve to the digitized points, while preserving intercanine and intermolar widths, and anterior and posterior arch depths.

To assess the curve fit, Pearson correlation coefficients were calculated between the 14 digitized incisal and occlusal landmarks that were used for curve construction (Fig 1). Because the method of least squares assumes that all variability is in the y coordinates, this means that the y_1 value of every digitized point with coordinates (x_1, y_1) is correlated to the y_2 value of the corresponding point on the curve (x_1, y_2). To allow comparisons with other studies, the residuals were calculated (difference, $y_2 - y_1$), as well as the mean square error (calculated by using the residuals), and root mean square values. Finally, the average perpendicular distance of the points to the curve (APDPTC) was calculated, with and without absolute values. During curve construction, the APDPTC was also used to select the optimal anterior curve (parabola or hyperbolic cosine function), based on the smallest mean APDPTC. This was done to accommodate slightly broader and more tapered anterior arch forms. All calculations were also performed for the curves resulting from the application of the model proposed by Noroozi et al[10] to the current patient sample, as mentioned below.

To assess the possible advantages of using 3 functions linked together, with a bilateral COG to optimally describe the lateral arch segments, the current model was compared with the work of Noroozi et al.[10] They introduced a model describing the human arch form using 1 function of the form $Y = AX^6 + BX^2$. Careful calculation of A and B allowed the maintenance of original intercanine and intermolar widths, as well as arch depth, while equally maintaining a close fit of the curve to the digitized points (Fig 4). Since the model proposed here is more complicated than the one they introduced, it was decided to apply the model of Noroozi et al[10] to this patient sample to ascertain whether the added complexity improved curve fit.

To establish the repeatability of the digitizing procedure, all points were redigitized for 10 randomly selected patients, rendering 140 consecutive measurements. The Wilcoxon signed rank test on the difference between the original and repeated x and y coordinates separately showed no significant differences (x coordinates: mean difference, 0 mm, SD, 0.011 mm; y coordinates: mean difference, 0 mm, SD, 0.011 mm).

Statistical analysis

All tests were performed with the Statistical Package for Social Sciences (SPSS for Windows, version 14, Chicago, Ill). Significance was predetermined at the 0.05 level of confidence. The comparison of boys and girls was made with either *t* tests or Mann-Whitney U tests, depending on the Levene test to confirm homogeneity of variance and the Shapiro-Wilk test to assess normality of the distribution.

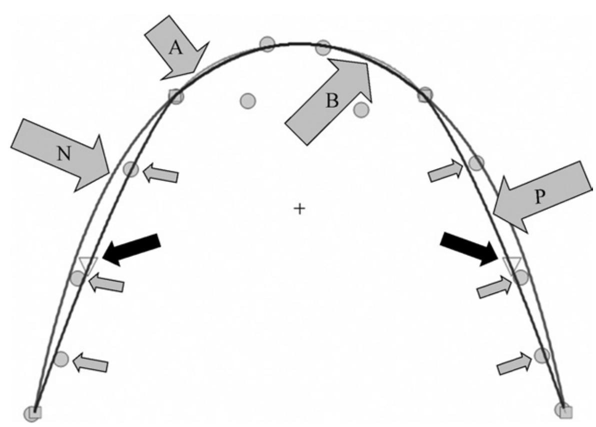

Fig 4. Functions used for curve fitting. Anterior functions: A. hyperbolic cosine function[9]: y = (−cosh(x/(ICW/2)) cosh(PAD +1)) + 1 + PAD)f, where *ICW* = intercanine width, *PAD* = posterior arch depth, and *f* = curve-fitting factor, calculated by FindGraph. B. parabola: y = i(fx^2+gx+h), where *f, g, h,* and *i* are curve-fitting factors, calculated by FindGraph. The curves were fitted by using following coordinates (x,y): ((−ICW/2),0); (0,AAD); ((ICW/2),0). Left and right lateral function: polynomial: y = fx^3+gx^2+hx, or alternatively y = x(fx^2+gx+h), where *f, g,* and *h* are curve-fitting factors, calculated by FindGraph. Curves were fitted by using following coordinates (x,y): ((−IMW/2), −PAD); ((−ICW/2),0); ((ICW/2),0); ((IMW/2), −PAD). Noroozi et al[10] originally defined formula of sixth-order polynomial function as:

$$y = 64\left(\frac{D_cW_m^2 - D_mW_c^2}{W_c^2W_m^2 - W_c^2W_m^6}\right)x^6 + 4\left(\frac{D_cW_m^6 - D_mW_c^6}{W_c^2W_m^6 - W_c^6W_m^2}\right)x^2$$

where (1) intercanine width (W_c) = distance between distal contact points of canines, (2) canine depth (D_c) = distance between contact of central incisors and line connecting distal contact points of canines, (3) intersecond molar width (W_m) = distance between distal contact points of second molars, (4) second molar depth (D_m) = distance between contact point of central incisors and line connecting distal contact points of second molars. In this project, parameters W_c, D_c, W_m, and D_m were adapted as follows: W_c = ICW; D_c = AADf (anterior arch depth, multiplied by curve-fitting factor f, mentioned above. This was done because contact points of central incisors, used to define factor D_c, were not digitized. This ensured that anterior arch depth was same for both current model and that os Noroozi et al.[10]); W_m = IMW; D_m = TAD. Parameters were inserted into formula for curve construction. Calculation of bilateral third curve-fitting point for polynomial function: 6 digitized points (*small grey arrows*) are used to calculate 2 extra points for lateral curve fitting with polynomial. By averaging absolute values of x coordinates of these points and y values, COG is computed for (right) lateral segment. Resulting point is mirrored around y-axis, rendering both lateral COG (*small black arrows*).

Table I. Intragroup comparisons

	Boys (n = 14)		*Girls (n = 16)*				
	Average	*SD*	*Average*	*SD*	*Mean difference*	*SD*	*P*
Intercanine width	25.54	1.64	25.85	1.39	−0.32	1.51	NS
Intermolar width	47.07	1.98	46.27	2.30	0.80	2.16	NS
Anterior arch depth	4.96	1.17	4.55	1.23	0.41	1.20	NS
Posterior arch depth	25.81	1.59	25.73	1.73	0.08	1.67	NS
Total arch depth	30.77	2.41	30.28	2.06	0.49	2.23	NS
Little's irregularity index	7.06	3.54	7.29	2.64	−0.22	3.09	NS

NS, not significant; SD pooled for sexes.

Table II. Curve parameters

	Average	*SD*	*Minimum*	*Maximum*
Intercanine width	25.71	1.49	21.48	28.54
Intermolar width	46.64	2.16	42.26	51.32
Anterior arch depth	4.74	1.20	2.62	7.01
Posterior arch depth	25.76	1.64	22.32	29.39
Total arch depth	30.51	2.21	25.69	34.92
Little's irregularity index	7.18	3.04	3.50	18.00

RESULTS

The results of the intragroup comparison are presented in Table I. Because no significant differences were found, the data for boys and girls were pooled for further analysis. The results for the combined data are summarized in Table II.

When comparing curve fit with Pearson correlation coefficients, an average coefficient of 0.994 was found (SD, 0.004) (Table III). The mean square error was found to be 1.80 mm^2 (SD, 1.12 mm^2), whereas the average root mean square value measured 1.28 mm (SD, 0.42 mm). The corresponding results for the sixth-order polynomial function as proposed by Noroozi et al[10] are listed in Table III.

As an alternative to the measures of vertical correspondence, the average perpendicular distance of the various digitized points to the constructed curve was calculated for both models (current model: average: 0.47 mm, SD, 0.16 mm) (Table III) in accordance with the study by BeGole.[21] To determine curve fit with the cubic spline analysis, she measured the length of a set of normals to the curve.

Finally, when computing the average signed perpendicular distance of the points to the curve, which indicates whether the constructed curves are generally too wide (positive values) or too small (negative values), a statistically (and clinically) insignificant average value of 0.04 mm was found (SD, 0.14 mm; $P > .05$ using 1-sample *t* test to test difference from zero). The value found for the model of Noroozi et al[10] was 0.5 mm (SD, 0.25 mm) (Table III).

DISCUSSION

A problem to overcome when attempting to describe the arch form of a patient with crowding or malposition is the proper orientation of the cluster of digitized points. Because slight asymmetries in the anteroposterior position of the first molars (or canines) can be present, using these teeth as the basis to construct the arch form can lead to arch canting or skewing: the constructed arch can bypass many points because 1 molar is located slightly more anteriorly than the other.[27] Sampson[22] studied arch shapes in 66 patients at the Center of Human Growth and Development at the University of Michigan, using conic sections (a family of curves including the circle, ellipse, parabola, and hyperbola that result from the intersection of a cone by a flat plane). The curves were constructed by using the first molar centroids as end points. He noticed that, in several patients, the major axis of the constructed ellipse was not vertical: the curve was skewed to the left or right. With the method of least squares in a Cartesian axis system to construct the above-mentioned arch forms (as was done here), it is not possible to skew the arch form to obtain an optimal fit. The procedure described in Figure 2 attempts to solve this problem, rotating the digitized points in analogy to the transformation matrix procedure of BeGole.[21] This allows for a more centered position of the resulting curve in relation to the digitized points, as was clearly evident on visual inspection.

When using a single mathematical function to describe the entire human arch form, maintaining intercanine and intermolar widths and anterior and posterior arch depths while maintaining a close fit to the digitized landmarks becomes increasingly difficult.[9] Nevertheless, exactly reproducing and controlling these measurements allows the practitioner to assess space conditions more accurately[10] and assures a closer congruence with the original arch form, which might promote future stability.[28] It was therefore decided to use the technique described by Hnat et al,[9] with 2

Table III. Curve fit results

	Combined function				Noroozi et al[10]			
	Average	*SD*	*Minimum*	*Maximum*	*Average*	*SD*	*Minimum*	*Maximum*
Correlation coefficient	0.994	0.004	0.984	0.999	0.985	0.009	0.960	1.000
PDPTC	0.47	0.16	0.26	0.84	0.70	0.26	0.28	1.40
Signed PDPTC	0.04	0.14	−0.16	0.49	0.50	0.25	0.08	1.27
Mean squared dist.	1.80	1.12	0.33	4.18	5.56	3.33	0.38	0.38
Root mean square value	1.28	0.42	0.57	2.05	2.24	0.76	0.61	3.62

PDPTC, Perpendicular distance of the points to the curve; dist, distance.

Table IV. Tooth-by-tooth analysis of normals

	Combined function			Noroozi et al[10]		
	Normals			*Normals*		
Tooth	*Average*	*SD*	*Total average (%)*	*Average*	*SD*	*Total average (%)*
36a	0.41	0.25	6.26	0.31	0.20	3.22
36b	0.35	0.26	5.29	0.67	0.42	6.91
35/75	0.44	0.31	6.75	1.27	0.85	13.00
34	0.59	0.42	8.93	0.98	0.72	10.02
33	0.45	0.48	6.85	0.48	0.50	4.92
32	0.73	0.82	11.16	0.81	0.89	8.33
31	0.24	0.19	3.68	0.24	0.19	2.51
41	0.24	0.19	3.69	0.24	0.20	2.46
42	1.03	0.89	15.63	1.19	0.94	12.24
43	0.48	0.47	7.30	0.47	0.47	4.84
44	0.46	0.42	6.96	0.95	0.68	9.75
45/85	0.47	0.35	7.12	1.17	0.71	12.01
46b	0.33	0.19	4.98	0.64	0.36	6.57
46a	0.35	0.22	5.38	0.32	0.20	3.25

a, Distobuccal/midbuccal cusp in the case of 2/3 cusp tips, respectively; b, the mesiobuccal cusp tip.

posterior and 1 anterior arch form strung together, to reconstruct the mandibular arch form. As an alternative, Noroozi et al[10] presented an efficient model whereby the human arch form is described with a single function (sixth-order polynomial) that complies with the requirements mentioned above, while obtaining a close fit to the digitized points. However, because their model consists of 1 function, it does not allow flexibility in the description of the lateral segments, as discussed below.

Contrary to the work of Hnat et al,[9] the transition between the anterior and posterior functions was not placed at the canine's distal anatomic contact point, because this landmark was frequently found to be displaced due to rotation. The canine cusp tip was chosen instead, because it was closer to the center of the tooth, resulted in slightly smoother arch forms, and facilitated the maintenance of intercanine width during curve construction. This made it possible to construct the arch form while respecting intercanine and intermolar widths, as well as anterior and posterior arch depths (Fig 4).

To allow for a proper description of the lateral segments, a method is required that forces the constructed curve through the averaged positions of the canine and the first molar (distobuccal or midbuccal) cusp tips, while allowing adjustment for lateral arch convexity; ideally, the curve should pass as close as possible to the points between these landmarks, without becoming too wavy. To overcome this problem, a bilateral COG was calculated (Fig 4). By fitting a third-order polynomial through the averaged positions of the canine and the first molar (distobuccal or midbuccal) cusp tips, and the bilateral COG, lateral curves can be described for broad and straight arch forms while exactly reproducing the initial curve parameters. In doing so, the lateral COG has a similar function as the knot points in the cubic spline function described by BeGole.[21]

Instead of calculating a separate COG for the left and right lateral segments, a mirrored COG was used. This was done to limit the influence of outliers on curve construction. Because these patients had crowding, it was feared that a lateral tooth displaced to the buccal or the lingual side would drag the curve too much outward or inward, respectively. There are basically 2 ways to overcome this problem: omit the outlier, or try to increase the number of points, lowering the weight of the outlier during the fitting procedure. Because the first option could introduce bias due to personal preferences (how does one define "outlier"?) and the number of points defining the lateral COG was already small, the second option was selected—increasing the number of points by mirroring them around the y-axis (Fig 3). This means that the arch forms rendered with this method were by definition bilaterally symmetric, even though mild asymmetry has been shown to occur naturally.[27] Nevertheless, in view of these reasons, the second option was considered to be more appropriate.

The resulting correlation coefficients were found to be surprisingly high and correspond well to previous studies; when using the generalized beta function to describe the human arch form, Braun et al[23] found an average mandibular correlation coefficient of 0.98 (SD, 0.1), whereas Noroozi et al,[20] applying the polynomial function $y = ax^6 + bx^2$, reported an average mandibular correlation coefficient of 0.97 (SD, 0.02). Both studies used well-aligned adult dentitions, including the second definitive molars in the analysis. Because in the late mixed dentition stage of dental development these teeth are frequently not erupted, it was decided not to include them in this curve-fitting procedure. Interestingly, when applying the model proposed by Noroozi et al[20] to this patient sample, slightly lower correlation coefficients were found compared with the currently proposed model (0.985 vs 0.994), with a slightly higher SD (0.009 vs 0.004) (Table III), although the differences were quite small.

The mean square error results allowed comparison with a study by Pepe.[19] She mentioned the mean square error values when fitting second-, fourth-, and sixth-degree polynomials to the (maxillary and) mandibular arch. The averaged mandibular results, calculated from table three in her study, were 4.30, 3.01, and 2.57 mm^2, respectively, compared with 1.80 mm^2 for the combined function in this study. The study of Pepe[19] had a small sample (7 patients) that might in part explain the difference. Also, she described adult dentitions with good occlusions. As reported in Table III, the corresponding values after applying the model of Noroozi et al[20] to this patient sample were higher.

Probably the easiest way to assess curve fit is to calculate the perpendicular distance of the various points to the constructed curve. This was proposed by BeGole[21] when investigating the applicability of the cubic spline function for the description of the human arch form. After constructing 27 maxillary curves, she performed a tooth-by-tooth, pairwise, segmental analysis, measuring the distance along a set of normals: lines perpendicular to a tangent constructed on the curve to minimize the distance from the exterior point to the curve.[21] In this experiment, an average perpen-

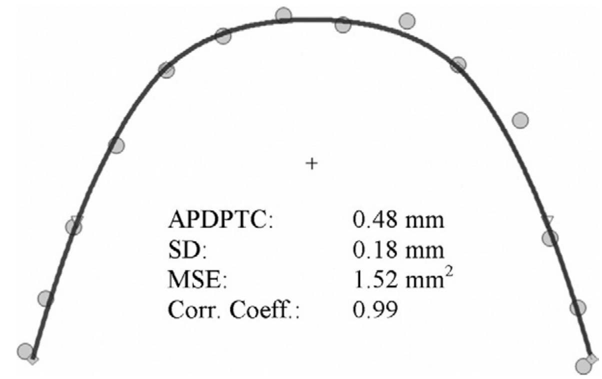

Fig 5. Example of curve fit for patient with average APDPTC and SD.

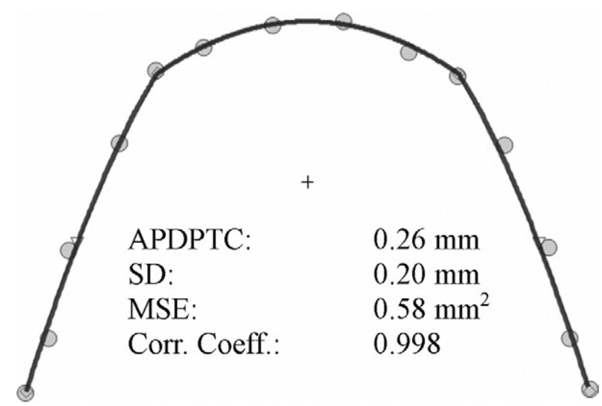

Fig 6. Example of curve fit for patient with lowest APDPTC.

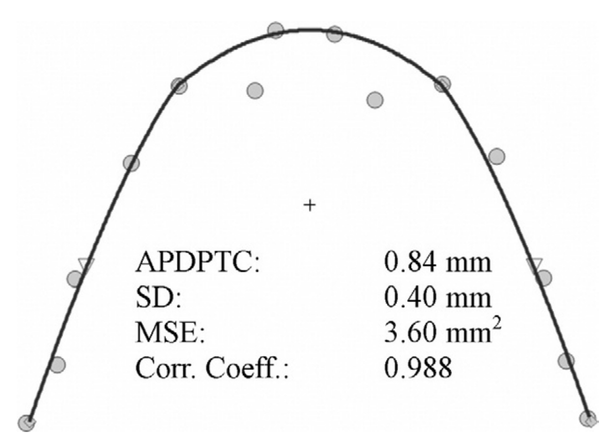

Fig 7. Example of curve fit for patient with highest APDPTC.

Table V. Error attibutable to left-right pairs

	Combined function					Noroozi et al[10]				
	Sum of normals		Average normal		Total average (%)	Sum of normals		Average normal		Total average (%)
	Average	SD	Average	SD		Average	SD	Average	SD	
31 + 41	0.49	0.38	0.24	0.19	7.40	0.48	0.38	0.24	0.19	4.88
32 + 42	1.76	1.60	0.88	0.80	26.93	2.00	1.73	1.00	0.86	20.20
33 + 43	0.93	0.95	0.47	0.47	14.23	0.95	0.96	0.48	0.48	9.58
34 + 44	1.05	0.64	0.52	0.32	15.97	1.93	1.28	0.96	0.64	19.42
35/75 + 45/85	0.91	0.50	0.46	0.25	13.95	2.44	1.49	1.22	0.75	24.56
36b + 46b	0.68	0.34	0.34	0.17	10.32	1.31	0.68	0.66	0.34	13.23
36a + 46a	0.77	0.41	0.35	0.16	11.21	0.63	0.40	0.49	0.21	8.13

a, Distobuccal/midbuccal cusp in the case of 2/3 cusp tips, respectively; b, the mesiobuccal cusp tip.

Table VI. Evaluation of error by segment

	Combined function					Noroozi et al[10]				
	Sum of normals		Average normal		Total average (%)	Sum of normals		Average normal		Total average (%)
	Average	SD	Average	SD		Average	SD	Average	SD	
Left lateral (36-34)	1.79	0.71	0.45	0.18	29.85	3.23	1.28	0.81	0.33	35.35
Front (33-43)	3.18	1.93	0.53	0.33	43.36	3.44	2.07	0.57	0.35	30.98
Right lateral (44-46)	1.61	0.58	0.40	0.15	26.80	3.08	1.30	0.77	0.33	33.67

dicular distance (or conversely, a mean normal length) of 0.45 mm was found (SD, 0.18 mm). Because the measured irregularity index was on average 7.2 mm (SD, 3.03 mm), this value can be considered small. These results are comparable with those of BeGole,[21] who found a mean normal of 0.59 mm (SD, 0.23 mm). The fact that she investigated curve fit using maxillary models could explain the slightly higher values reported, because previous studies mentioned slightly better results for curve fit in mandibular vs maxillary arches.[19] From the tooth-by-tooth, pairwise, segmental analysis, presented in Tables IV through VI, it is evident that the largest contribution to the fitting error originates from the lateral incisors, which were frequently found to be lingually displaced. For the other teeth, the error seems to be evenly distributed.

Examples of constructed curves and their resulting values for curve fit are given in Figures 5 through 7.

Comparing this proposed model with that of Noroozi et al,[20] the advantages of a lateral COG to describe the buccal segments can be clearly seen. Tables V and VI list the error attributable to left-right pairs and the segmental error, respectively. In both tables, it is evident that this proposed model consisting of 3 functions and using the lateral COG maintains a much closer fit to the lateral teeth in comparison with the single sixth-order polynomial function of Noroozi et al.[20] The normal lengths for the lateral teeth with their model are consistently higher.

Averaging the signed values for the perpendicular distance of the points to the curve highlights the relative distribution of the points in relation to this curve. For the current model, the nonsignificant average signed distance of 0.04 mm (SD, 0.14 mm) (Table III) indicates that the constructed curves were on average neither too broad nor too small. For the model of Noroozi et al,[20] the corresponding value was 0.5 mm; this means that the constructed curves were on average (slightly) too broad in this patient sample. Furthermore, the SD of 0.25 mm, compared with 0.14 mm for the current model, indicates a (slightly) smaller dispersion of the digitized points around the constructed curve in the current model.

CONCLUSIONS

Using a polynomial function laterally between the molar distobuccal or midbuccal cusp tip and the canine tip, combined with a parabola or hyperbolic cosine function for the frontal region (between the canine tips), allows the practitioner to respect intercanine and intermolar widths, and anterior and posterior arch depths, while accurately describing the mandibular arch forms in late mixed dentition patients with crowding. To allow for a proper description of the lateral arch form, a symmetrical lateral COG was calculated, and a third-order polynomial was fitted through the calculated point and the cusp tips. The resulting values for correlation coefficients, mean square error, and average perpendicular distance of the points to the curve suggest that the method is successful at describing the mandibular arch forms of mixed dentition patients with crowding, and it could be a useful adjunct to treatment planning.

I thank R. De Rycke, M. Couvreur, P. Raymaeckers, K. Houthoofd, the Scientific Committee of the Society of Flemish Dentists, S. Vasilyev for his programming skills, E. BeGole for her advice, support, and proofreading of the manuscript, and my wife.

REFERENCES

1. Allais D, Melsen B. Does labial movement of lower incisors influence the level of the gingival margin? A case-control study of adult orthodontic patients. Eur J Orthod 2003;25:343-52.
2. Housley JA, Nanda RS, Currier GF, McCune DE. Stability of transverse expansion in the mandibular arch. Am J Orthod Dentofacial Orthop 2003;124:288-93.
3. Melsen B, Dalstra M. Distal molar movement with Kloehn headgear: is it stable? Am J Orthod Dentofacial Orthop 2003;123:374-8.
4. Brennan MM, Gianelly AA. The use of the lingual arch in the mixed dentition to resolve incisor crowding. Am J Orthod Dentofacial Orthop 2000;117:81-5.
5. Sheridan JJ. Air-rotor stripping. J Clin Orthod 1985;19:43-59.
6. Del Santo M Jr, Guerrero CA, Buschang PH, English JD, Samchukov ML, Bell WH. Long-term skeletal and dental effects of mandibular symphyseal distraction osteogenesis. Am J Orthod Dentofacial Orthop 2000;118:485-93.
7. Kirschen RH, O'Higgins EA, Lee RT. The Royal London Space Planning: an integration of space analysis and treatment planning: Part I: Assessing the space required to meet treatment objectives. Am J Orthod Dentofacial Orthop 2000;118:448-55.
8. Germane N, Lindauer SJ, Rubenstein LK, Revere JH Jr, Isaacson RJ. Increase in arch perimeter due to orthodontic expansion. Am J Orthod Dentofacial Orthop 1991;100:421-7.
9. Hnat WP, Braun S, Chinhara A, Legan HL. The relationship of arch length to alterations in dental arch width. Am J Orthod Dentofacial Orthop 2000;118:184-8.
10. Noroozi H, Djavid GE, Moeinzad H, Teimouri AP. Prediction of arch perimeter changes due to orthodontic treatment. Am J Orthod Dentofacial Orthop 2002;122:601-7.
11. Hussels W, Nanda RS. Effect of maxillary incisor angulation and inclination on arch length. Am J Orthod Dentofacial Orthop 1987;91:233-9.
12. O'Higgins EA, Kirschen RH, Lee RT. The influence of maxillary incisor inclination on arch length. Br J Orthod 1999;26:97-102.
13. Braun S, Hnat P, Kusnoto B, Hnat TW. A new accurate approach to the anterior ratio with clinical applications. Part 1: a computer program. Am J Orthod Dentofacial Orthop 1999;115:368-72.
14. Sheridan JJ. Air-rotor stripping. J Clin Orthod 1985;19:43-59.
15. Germane N, Staggers JA, Rubenstein L, Revere JT. Arch length considerations due to the curve of Spee: a mathematical model. Am J Orthod Dentofacial Orthop 1992;102:251-5.
16. Braun S, Hnat WP, Johnson BE. The curve of Spee revisited. Am J Orthod Dentofacial Orthop 1996;110:206-10.
17. Jones ML, Richmond S. An assessment of the fit of a parabolic curve to pre- and post-treatment dental arches. Br J Orthod 1989;16:85-93.
18. Currier JH. A computerized geometric analysis of human dental arch form. Am J Orthod 1969;56:164-79.
19. Pepe SH. Polynomial and catenary curve fits to human dental arches. J Dent Res 1975;54:1124-32.
20. Noroozi H, Nik TH, Saeeda R. The dental arch form revisited. Angle Orthod 2001;71:386-9.
21. BeGole EA. Application of the cubic spline function in the description of dental arch form. J Dent Res 1980;59:1549-56.
22. Sampson PD. Dental arch shape: a statistical analysis using conic sections. Am J Orthod. 1981;79:535-48.
23. Braun S, Hnat WP, Fender DE, Legan HL. The form of the human dental arch. Angle Orthod 1998;68:29-36.
24. Lestrel PE, Takahashi O, Kanazawa E. A quantitative approach for measuring crowding in the dental arch: Fourier descriptors. Am J Orthod Dentofacial Orthop 2004;125:716-25.
25. Brennan MM, Gianelly AA. The use of the lingual arch in the mixed dentition to resolve incisor crowding. Am J Orthod Dentofacial Orthop 2000;117:81-5.
26. Little RM. The irregularity index: a quantitative score of mandibular anterior alignment. Am J Orthod 1975;68:554-63.
27. Maurice TJ, Kula K. Dental arch asymmetry in the mixed dentition. Angle Orthod 1998;68:37-44.
28. Felton JM, Sinclair PM, Jones DL, Alexander RG. A computerized analysis of the shape and stability of mandibular arch form. Am J Orthod Dentofacial Orthop 1987;92:478-83.